Journal of Basic and Applied Engineering Research
Print ISSN: 2350-0077; Online ISSN: 2350-0255; Volume 1, Number 14; October-December 2014 pp. 113-116

http://www.krishisanskriti.org/jbaer.html

Static Deformation of a Two-Dimensional Half-Space Due to a Vertical Tensile Fault at an Arbitrary Location

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Abstract—*The purpose of the present note is to study the deformation of a uniform half-space caused by a vertical tensile fault placed at an arbitrary location. The Airy stress function for a long tensile fault placed at an arbitrary position* (d_1, d_2) *in a homogeneous, isotropic, uniform half-space is obtained. Closed-form analytical expressions of stresses are derived using the Airy stress function approach. Then, the variation of the stress field is studied numerically. It is noted that the parameters* d_1 *and* d_2 *occur explicitly in the final expressions of stresses.*

1. INTRODUCTION

Steketee [8,9] laid the foundation of elasticity theory of dislocations in the field of seismology. Since then, numerous studies have been done in this field. Tensile fault representation has various geophysical applications such as mine collapse, fluid driven cracks and modeling of the deformation field due to a dyke injection in the volcanic region.

However, the studies related to the modelling of the deformation field due to a tensile fault are scarce than due to a shear fault. The problem of a tensile fault in a uniform half-space has been studied by several researchers. Maruyama [3], Davis [2], Yang and Davis [10], Bonafede and Danesi [1], Singh and Singh [6], Singh et al. [7]. Singh and Singh [6] obtained the stress and displacement fields for a uniform half-space caused by a tensile fault when the upper edge of the fault is placed at origin. Singh et al. [7] extended their work by moving the upper edge of the fault in vertical direction only. The aim of the present paper is to find stress fields when the upper edge of the fault is moved both in horizontal as well as vertical direction. The depths d_1 and d_2 occur explicitly in the solution. The variation of the stress fields with the distance from the fault and with depth is studied with the help of figures.

2. THEORY

Consider the Cartesian coordinate system (x_1, x_2, x_3) with the x_3-axis vertically downwards. Consider a homogeneous, isotropic, perfectly elastic uniform half-space. A long inclined tensile fault is placed at an arbitrary position (d_1, d_2), δ is the dip angle and b is the width of the fault.

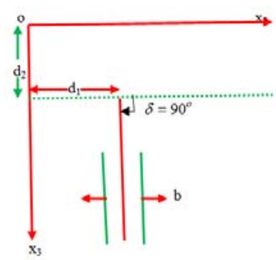

Fig. 1: Geometry of a vertical tensile fault

Under plane strain problem, the displacement components u_i $(i = 1,2,3)$ are independent of x_1 so that $\partial/\partial x_1 = 0$. The stresses in terms of the Airy stress function U are given by

$$\tau_{22} = \partial^2 U/\partial x_3^2, \ \ \tau_{33} = \partial^2 U/\partial x_2^2, \ \ \tau_{23} = -\partial^2 U/\partial x_2 \partial x_3 \tag{1}$$

As shown by Singh and Garg [5], the Airy stress function U_0 for a line source parallel to x_1-axis passing through the point $(0, 0, h)$ in an infinite medium can be expressed in the form:

$$U_0 = \int_0^\infty \Big[\big(L_0 + M_0 k \left| x_3 - h \right| \big) \sin kx_2$$

$$+\left(P_0+Q_0k\left|x_3-h\right|\right)\cos kx_2\Big]k^{-1}e^{-k\left|x_3-h\right|}dk \quad (2)$$

where the source coefficients L_0, M_0, P_0 and Q_0 are independent of *k*.

For a line source parallel to x_1-axis acting at the point $(0,0,h)$ of the half-space $x_3 \geq 0$, a suitable solution of the biharmonic equation

$$\nabla^2\nabla^2 U = 0 \quad (3)$$

is of the form

$$U = U_0 + \int_0^\infty \Big[\left(L + Mkx_3\right)\sin x_2$$

$$+\left(P + Qkx_3\right)\cos kx_2\Big]k^{-1}e^{-kx_3}dk \quad (4)$$

where U_0 is given by equation (2) and L, M, P and Q are unknowns to be determined from the boundary conditions.

Following Rani et al. [4] and Singh and Singh [6], we obtain the Airy stress function for an inclined tensile fault of finite width L and infinite length placed at an arbitrary position (d_1, d_2) (where d_1 is the distance from x_3-axis and d_2 is the distance from x_2-axis) in a uniform half-space $x_3 \geq 0$;

$$U = \left[\mu b/2\pi(1-\sigma)\right]\Big\{\left(s - X_2\cos\delta - X_3\sin\delta\right)$$

$$\times\log_e\left(S/R\right) + 2x_3\Big[X_2\left(X_2\sin\delta + d_2\cos\delta\right)$$

$$+x_3X_3^{'}\sin\delta\Big]S^{-2} + 2sx_3$$

$$\times\Big[\left(X_3^{'}\sin\delta - X_2\cos\delta\right)\sin\delta - d_2\Big]S^{-2}\Big\}\Big\| \quad (5)$$

where

$$X_2 = x_2 - d_1,\ X_3 = x_3 - d_2,\ X_3^{'} = x_3 + d_2,$$

$$R^2 = \left(X_2 - s\cos\delta\right)^2 + \left(X_3 - s\sin\delta\right)^2,$$

$$S^2 = \left(X_2 - s\cos\delta\right)^2 + \left(X_3^{'} + s\sin\delta\right)^2,$$

$$f(s) = f(L) - f(0). \quad (6)$$

Using equations (1) and (5), we obtain the following expressions for the stress components (for $\delta = 90^o$);

$$\tau_{22} = \left[\mu b/2\pi(1-\sigma)\right]\Big\{\left(X_3 - s\right)R^{-2} - \left(3x_3 + d_2 + s\right)S^{-2}$$

$$+2X_2^2\left(X_3 - s\right)R^{-4} + 2\Big[\left\{x_3^2 + \left(d_2+s\right)^2\right\}\left\{X_3^{'} + s\right\}$$

$$-2X_2^2\left(d_2+s\right)\Big]S^{-4} + 16X_2^2x_3\left(d_2+s\right)\left(X_3^{'}+s\right)S^{-6}\Big\}\Big\| \quad (7)$$

$$\tau_{23} = \left[-\mu bX_2/2\pi(1-\sigma)\right]\Big\{-\left(R^{-2}+S^{-2}\right) + 2X_2^2R^{-4}$$

$$+2\Big[x_3^2 + \left(d_2+s\right)^2 - 4x_3\left(d_2+s\right)\Big]S^{-4}$$

$$+16X_2^2x_3\left(d_2+s\right)S^{-6}\Big\}\Big\| \quad (8)$$

$$\tau_{33} = \left[\mu b/2\pi(1-\sigma)\right]\Big\{\left(X_3 - s\right)\left(R^{-2} - S^{-2}\right)$$

$$-2X_2^2\left(X_3 - s\right)\left(R^{-4} - S^{-4}\right)$$

$$+4x_3\left(d_2+s\right)\left(X_3^{'}+s\right)S^{-4}$$

$$-16X_2^2x_3\left(d_2+s\right)\left(X_3^{'}+s\right)S^{-6}\Big\}\Big\| \quad (9)$$

It has been verified that the stress components given in equations (7)-(9) satisfy the traction-free boundary conditions:

$$\tau_{23} = \tau_{33} = 0 \text{ at } x_3 = 0. \quad (10)$$

It has also been verified that on taking $d_1 = d_2 = 0$ in equations (7)-(9), the results of Singh and Singh [6] are obtained and on taking $\delta = 90^o$, $d_1 = 0$, $d_2 = d$ in equations (7)-(9), the results of Singh et al. [7] are obtained as a particular case.

3. NUMERICAL RESULTS AND DISCUSSION

To examine the effect of the depths d_1 and d_2 where the upper edge of the fault of finite width L is placed in a uniform half-space on the variation of the stress field, we assume that $\sigma = 0.25$ and put $s_1 = 0$, $s_2 = L$.

For numerical computations, we define the following dimesionless quantities:

$$Y = x_2/L,\ Z = x_3/L,\ D_i = d_i/L,\ P_{ij} = \left(\pi L/\mu b\right)\tau_{ij} \quad (11)$$

Figs. 2 (a,b,c) show the variation of the dimensionless normal stress P_{22} with the dimensionless distance from the fault at $Z = 0.1$, 0.5, 1.0 respectively for dimensionless depths $D_1 = D_2 = 0$, 0.5 and 1.0. It is noticed that as the value of Z increases, P_{22} converges to zero faster. The variation of P_{22} is smooth for each value of Z, D_1 and D_2.

Figs. 3 (a,b,c) show the variation of the dimensionless normal stress P_{33} with the dimension-

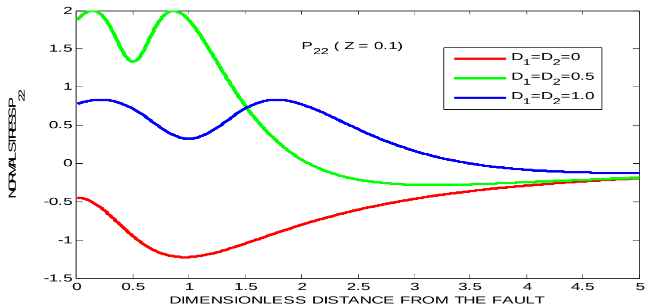

Fig. 2(a) Variation of the normal stress P_{22} with the dimensionless distance from the fault

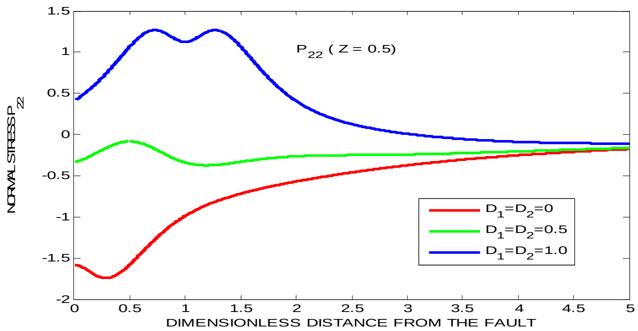

Fig. 2(b) Variation of the normal stress P_{22} with the dimensionless distance from the faul

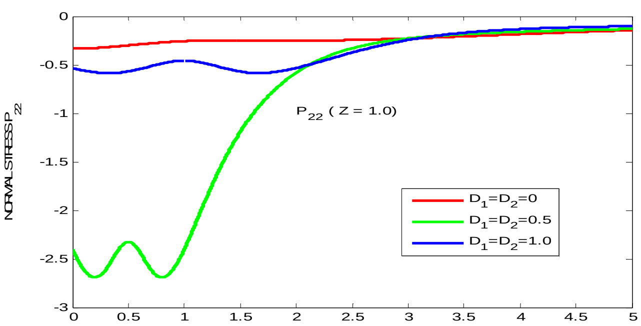

Fig. 2(c) Variation of the normal stress P_{22} with the dimensionless distance from the fault

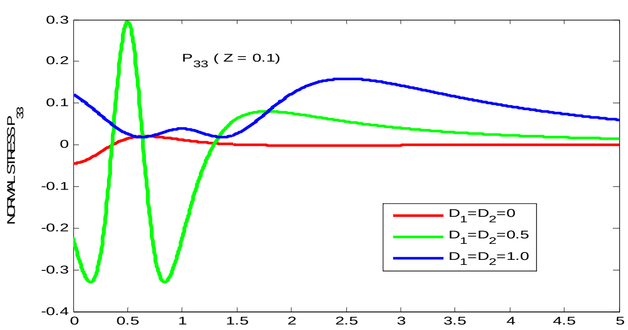

Fig. 3(a) Variation of the normal stress P_{33} with the dimensionless distance from the fault

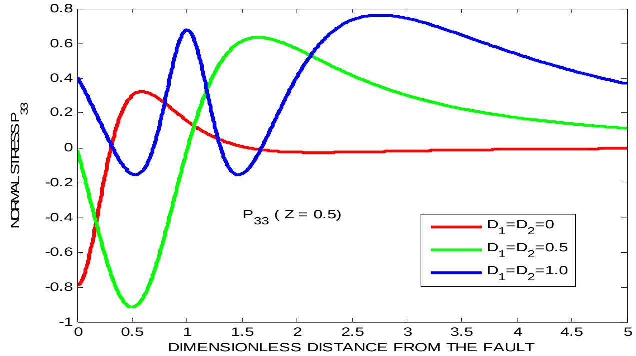

Fig. 3(b) Variation of the normal stress P_{33} with the dimensionless distance from the fault

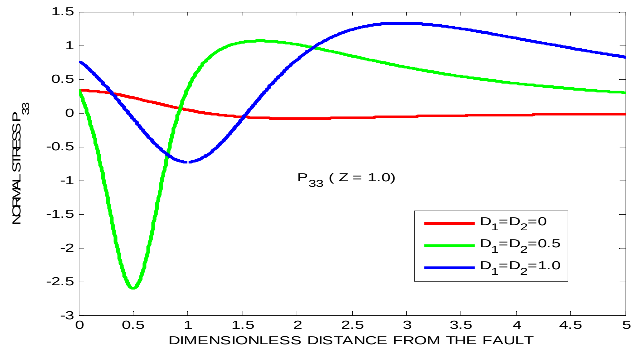

Fig. 3(c) Variation of the normal stress P_{33} with the dimensionless distance from the fault

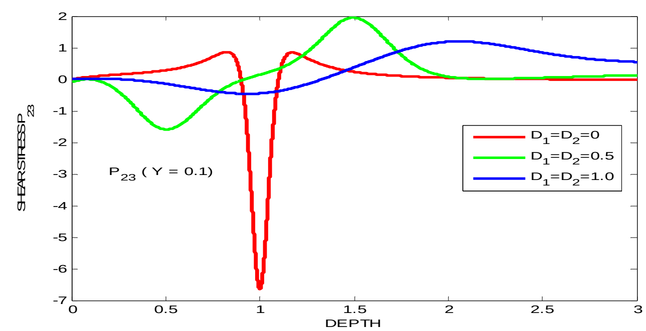

Fig. 4(a) Variation of the shear stress P_{23} with the depth from the fault

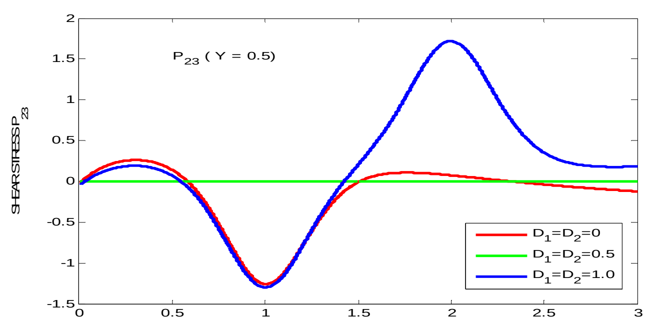

Fig. 4(b) Variation of the shear stress P_{23} with the depth from the fault

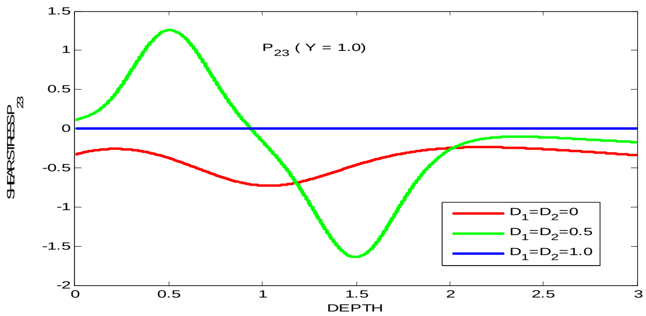

Fig. 4(c) Variation of the shear stress P_{23} with the depth from the fault

-less distance from the fault at $Z = 0.1, 0.5, 1.0$ respectively for dimensionless depths $D_1 = D_2 = 0$, 0.5 and 1.0. In this case, as we move away from the fault, P_{33} becomes zero at origin $(D_1 = D_2 = 0)$ for each value of Z. P_{33} assumes both positive as well as negative values for each value of D_1 and D_2. As Y tends to infinity, P_{33} converges to zero.

Figs. 4 (a,b,c) show the variation of the dimensionless shear stress P_{23} with the dimensionless depth at $Y = 0.1, 0.5, 1.0$ for dimensionless depths $D_1 = D_2 = 0$, 0.5 and 1.0. It is noticed that P_{23} becomes zero when value of Y coincides with values of D_1 and D_2. The variation of P_{23} with depth is smooth for all the cases. As Z tends to infinity, P_{23} converges to zero.

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